Learning mathematics from erroneous example in individual and collaborative setting: is it effective to facilitate students' mathematical disposition?

IOP Conf. Series: Journal of Physics: Conf. Series **1320** (2019) 012097 doi:10.1088/1742-6596/1320/1/012097

Learning mathematics from erroneous example in individual and collaborative setting: is it effective to facilitate students' mathematical disposition?

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Abstract. Learning mathematics from an erroneous example—worked example contains error—in individual and collaborative setting could be potential to facilitate students' mathematical disposition through activities such as find, explain, and correct the errors as well as solve a problem similar to the erroneous example. This study was intended to examine the effectiveness of learning mathematics from the erroneous example in individual and collaborative setting with regards to students' mathematical disposition. This quasi-experimental research with non-equivalent group pre-test and post-test design employed a 30 items-questionnaire to gather data of mathematical disposition. The participants were 45 students of grade 11th in the one of public senior high school in Yogyakarta. The collected data of students' mathematical disposition then were examined by using paired samples t-test, one sample t-test, and one-proportion z-test. This study revealed that learning mathematics from erroneous example could not significantly facilitate students' mathematical disposition in both individual and collaborative setting. The possible reasons behind these findings are discussed.

1. Introduction

One of the crucial skills that students should acquire throughout mathematics learning in order to deal with 21st-century citizenship and careers is problem-solving [1]. There are a lot of factors that may contribute to influence students' problem skills. One of those factors is mathematical disposition [2]. Mathematical disposition can be understood as a pattern of behaviour and mathematical cognitive habits of the students in learning mathematics and usually determined by how well they motivate themselves and persist in dealing with the difficulties [3]. Furthermore, according to the National Council of Teachers of Mathematics (NCTM) [4], mathematical disposition reflects students' confidence, willingness to explore the alternative methods of problem-solving, perseverance, interest towards mathematical tasks and the tendency for reflecting and monitoring their own thought and performance. The positive mathematical disposition is important to be developed by students as it can support them to be successful in learning mathematics as well as face up the diverse difficulties in daily life [5]. However, students' mathematical disposition is not facilitated maximally yet in mathematics learning [6–8].

Considering the importance of facilitating students' mathematical disposition as well as existed mathematics learning today, such an urgent situation to look for an appropriate strategy to create

mathematics learning that facilitating students' mathematics disposition effectively. One of the potential strategies of mathematics learning that may support students in developing their mathematical disposition is presenting erroneous examples—step-by-step problem solution that contains error—to students. Through erroneous example, students are facilitated to do activities such as detect, explain, and correct the existed error and after that solve a similar problem to the erroneous example [9]. Presenting erroneous example is considered as one of the potential strategies of mathematics learning for facilitating students' mathematical disposition in three ways as follow [10].

- Detecting error in erroneous example provides students the opportunity to understand alternative ways—including the error one—of problem-solving and may help students to develop their attitude of monitoring and reflecting their thought or performance as well as motivating students in learning mathematics.
- Erroneous example facilitates students to think and explore an alternative strategy of problem-solving, investigate structure and interconnection among each step of problem-solving, and discover the essential concept or idea within the erroneous example.
- Detecting, explaining, and correcting the error can promote the growth of self-explanation.

According to the previous research related to the implementation of an erroneous example in mathematics learning [11,12], the erroneous example can be presented in the individual or collaborative setting of learning. In the individual setting, the teacher facilitates students to accomplish a certain task, understand learning competencies, or achieve academic goal individually based on their own knowledge, understanding, or ability without considering any point of view of another students [13–15]. It seems that through individual learning students are encouraged to be independent in acquiring the knowledge or skills. In contrast to the individual setting, MacGregor argues collaborative learning facilitates students to solve a problem or attain a common academic goal by working together in groups [16]. In addition, the collaborative setting provides an opportunity for students to interact with all group member and help each other in order to discuss and understand a concept, discuss and compare every different opinion that may rise up from group member towards what they learn, and evaluate the result of discussion [17]. Both individual and collaborative setting may foster the potential of presenting an erroneous example to facilitate students in developing their mathematical disposition. However, this claim still needs to be confirmed with empirical data since the effectiveness of presenting an erroneous example to students in an individual and collaborative setting as an opportunity to facilitate students' mathematical disposition has not been studied before. Therefore, this study is aimed at examining the effectiveness of learning mathematics from the erroneous example in individual and collaborative setting viewed from students' mathematical disposition.

2. Literature Review

2.1. Learning mathematics from an erroneous example

Presenting students with an erroneous example for some mathematics teacher is considered as a strategy of learning that should be avoided since it may lead students to adopt the existed error in the erroneous example or increase students' misconception [9,18]. However, there is no sufficient evidence to believe this one [9,19]. Instead, previous studies found that learning mathematics from erroneous example facilitates students to develop their conceptual understanding [20], procedural understanding [12,21], metacognitive skills [22], and problem-solving skills. Moreover, presenting students with an erroneous example would make them have a positive attitude towards errors [12] and improve their self-efficacy [23].

Two things that should be considered by the teacher before presenting an erroneous example to students are the topic of learning and design of erroneous example. Previous studies [e.g. 24] suggest that erroneous example ideally presents common misconceptions or error made by students when they learn to solve a particular problem. It means that an erroneous example is desirable to be implemented in the topic of learning in which students have common misconceptions or errors on it. Nevertheless, a

study of an erroneous example conducted by Große and Renkl [21] does not in line with that suggestion. Therefore, it seems that an erroneous example is possible to be presented in all topic of mathematics learning by considering the learning objective that is going to be achieved. In designing erroneous example, a teacher can follow the steps of creating an erroneous example proposed by McGinn, Lange, and Booth [9], namely (1) determine the objective of presenting erroneous and common misconceptions or errors made by students, (2) choose misconceptions or errors that are going to be included in erroneous example, (3) create an erroneous example, (4) create a self-explanation prompts and (5) create a problem similar to the erroneous example.

2.2. Learning mathematics in an individual and collaborative learning

In the individual setting of learning, students should work on a task individually and learn collaboratively when the collaborative setting of learning is applied. Learning in individual and collaborative setting may be both beneficial and unbeneficial for students. For some situations, learning individually is more beneficial for students rather than learning collaboratively. This statement is supported by Bryson [25] who argues that when students learn collaboratively, there is a tendency that they will spend their time on chatting about the irrelevant topic with their task and neglect that task. On the other hand, when the individual setting is implemented, students tend to accomplish their task or ask the teacher for help when they have difficulties. But then, learning individually may be a source of difficulty for students that can lead students' interest to learn is decreased.

Meanwhile, learning mathematics in collaborative setting gives an opportunity for students to (1) develop their critical thinking skills, analytical thinking skills, communication skills, and collaborative skills, and (2) value and appreciate the another students' ideas, technics, and methods of problem-solving [26]. Furthermore, according to Kocak, Bozan, and Isik [27], when students' learn mathematics collaboratively, they are facilitated to be more confident, overcome their fear of making mistakes, reduce their anxiety towards mathematics, learn to respect with the ideas of other group members. It seems that the collaborative setting supports students to behave in positive ways toward mathematics.

2.3. Students' mathematical disposition

Students' mathematical disposition manifests students' response towards mathematical problem, perception about themselves as mathematical learners, persistence in attempting to understand a problem, procedure, concept, or other aspects of mathematics and willingness to collect a set of mathematics problems and attempt to solve those problems to help them to construct mathematical ideas [28]. In addition, students' mathematical disposition also can be understood as students' point of view towards the advantages and applications of mathematics in their activities and subcultures [28]. Furthermore, one of five categories of aptitude that should be mastered by students in order to develop their mathematical disposition is self-regulatory skills that cover skills relating to the self-regulation of one's cognitive and volitional process [30]. Self-regulatory skills also related to students' belief in a positive way towards their own capabilities and satisfaction with the other students' efforts [29].

Mathematical disposition is formed by some components. Watson [30] states that components of mathematical disposition are (1) self-efficacy—belief towards one's capabilities to use mathematics properly and effectively, (2) belief towards advantages and applications of mathematics, and (3) perseverance—belief towards the importance of perseverance as the determination of successfulness in solving a mathematics problem. Watson's statement is supported by Feldhaus [31] who argues that mathematical disposition covers attitude and belief toward mathematics, mathematical self-efficacy, and self-belief towards the role of perseverance in solving a mathematics problem. According to Philipp, attitude reflects one's behaving, feeling, or thinking towards a certain thing [32]. Based on the explanation above related to students' mathematical disposition, it can be inferred that students' mathematical disposition refers to students' point of view or tendency to (1) regulate their self throughout the learning process, (2) persevere and think creatively in dealing with difficulties, solving

mathematics problem, and understanding mathematical objects, and (3) appreciate the advantages and applications of mathematics in their daily life. Thus, from this definition of students' mathematical disposition, it can be derived aspects of mathematical disposition, namely (1) self-regulation, (2) creativity and perseverance, and (3) appreciation towards the advantages and applications of mathematics.

3. Methods

3.1. Participants

The entire population of this study was 11th graders of Mathematics and Science Program (MIPA) in one of the senior public schools in Yogyakarta city who were novices in the application of the first derivative. The research samples were two classes—MIPA 8 and MIPA 9—which respectively consist of 25 and 20 students. Students of the class MIPA 8 learned the application of the first derivative through erroneous example worksheets in an individual setting and the other one in a collaborative setting. In the collaborative setting, students were randomly assigned to small groups of 4-5 students.

3.2. Instrument

This is quasi-experimental research with pre-test post-test non-equivalent group design in which a mathematical disposition questionnaire employed to gather the data. This questionnaire covers three aspects of mathematical disposition, namely: self-regulation, creativity and perseverance, and appreciation towards advantages and applications of mathematics. These three aspects were elaborated into 10 indicators based on literature review [4,33,34], as can be seen in Table 1. From these indicators can be derived 30 statement items—16 positive statements and 14 negative statements—with four options of Likert scale 1 to 4 to categorise students' mathematical disposition.

Table 1. The aspects and indicators of mathematical disposition

- **Self-regulation**
  - Believe in the own ability to use mathematics well and effectively
  - Be able to regulate emotions and motivation in learning mathematics
  - Confident in dealing with a difficult mathematics problem
  - Confident in conveying ideas
  - Monitor and reflect on the own thoughts and performances
- **Creativity and perseverance**
  - Flexible in exploring mathematical ideas and try alternative methods to solve a mathematics problem
  - Look for strategies in solving a mathematics problem that can significantly increase the chances of getting a solution
  - Eager to strive to understand problems, procedures, concepts, or other aspects related to mathematics
  - Gather additional learning materials that can help in understanding material and reconstructing the important ideas
- **Appreciation towards advantages and applications of mathematics**
  - Believe in the advantages and applications of mathematics in daily life

3.3. Procedure

At the start of the study, the students completed a mathematical disposition questionnaire. One the next four meetings of learning, they learn the application of the first derivative—optimization of functions, determination of tangent and a normal line, determination of monotonicity and concavity of curve of the functions—through erroneous examples worksheets (see Figure 1). In each meeting of learning, before students learn with erroneous examples, the teacher gave a brief instruction in order to introduce the terminologies and basic concepts that students need to know. Soon after students learn about the basic concepts, they work with erroneous example worksheet. At the end of the study, again, students were asked to complete the mathematical disposition questionnaire.

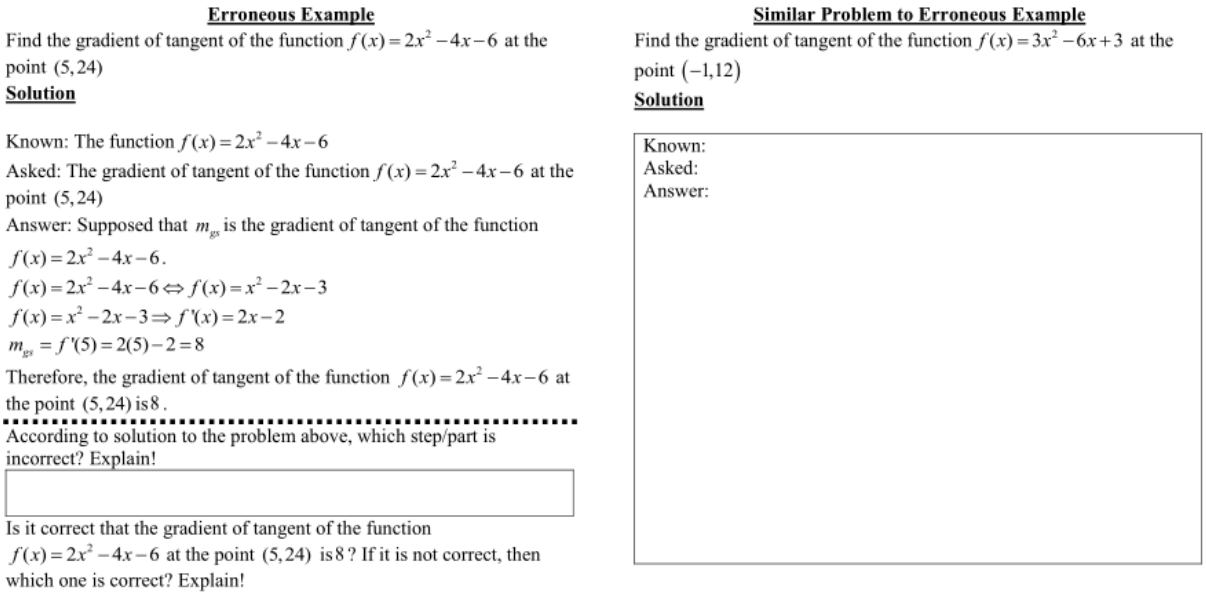

Figure 1. The example of the erroneous example and problem similar to the erroneous example that was developed in this study

3.4. Data analysis

The gathered data of students' mathematical disposition were analysed descriptively and inferentially by using the R software. Unfortunately, because of one student of the class MIPA 8 and five students of the class MIPA 9 were not filled the questionnaire before intervention and one student of the class MIPA 9 was not fully filled the questionnaire before intervention, just 25 data of class MIPA 8 and 20 data of class MIPA 9 students' mathematical disposition that included into analysis. First and foremost, the normality of the data was investigated before doing further analysis. The result of the Shapiro-Wilk test indicated that the data is normally distributed. Therefore, a parametric test was employed in this study.

Learning mathematics from erroneous example in individual and collaborative setting is said to be effective to facilitate students' mathematical disposition when these three criteria are satisfied, i.e., (1) there is students' mathematical disposition mean score improvement, (2) mean score of students' mathematical disposition after intervention is greater than 82.5 and (3) there are more than 70% of students whose mathematical disposition score is greater than 82.5 at the after intervention. These criteria were confirmed by using paired samples t-test, one sample t-test, and one-proportion z-test at a significance level of 0.05. If learning mathematics from the erroneous example in both individual and collaborative setting are effective, they will be analysed further to determine whether learning mathematics from the erroneous example in collaborative learning is more effective than an individual setting.

4. Result and Discussion

4.1. Descriptive result

The collected data of students' mathematical disposition score were analysed descriptively first to describe the mean and standard deviation of the data (see Table 2). The descriptive analysis revealed that the mean of the score of students' mathematical disposition in an individual setting increased by 1.16. On the contrary, the mean of students' mathematical disposition score in a collaborative setting decreased by 0.70.

Table 2. Description of the data on students' mathematical disposition score

Description	Individual Setting (MIPA8)		Collaborative Setting (MIPA 9)	
	Before Intervention	After Intervention	Before Intervention	After Intervention
Mean	85.04	86.20	82.20	81.50
Standard deviation	7.17	6.38	8	9.42

4.2. Students' mathematical disposition based on the categorisation of mathematical disposition score

This study also found that based on the data from the questionnaire at the before and after the intervention, more than a half of students in the individual setting have a high mathematical disposition (see Table 3). Meanwhile, the majority of students' mathematical disposition scores in a collaborative setting were in the medium category. Furthermore, there was a student in a collaborative setting—at before and after intervention—that his mathematical disposition score was at the low category.

Table 3. Categorisation of the mathematical disposition score

Category	Individual Setting (MIPA 8)		Collaborative Setting (MIPA 9)	
	Before Intervention	After Intervention	Before Intervention	After Intervention
Very High ($97.5 < x$)	1 (4%)	1 (4%)	1 (5%)	2 (10%)
High ($82.5 < x \leq 97.5$)	14 (56%)	17 (68%)	8 (40%)	6 (30%)
Medium ($67.5 < x \leq 82.5$)	10 (40%)	7 (28%)	10 (50%)	11 (55%)
Low ($52.5 < x \leq 67.5$)	0 (0%)	0 (0%)	1 (5%)	1 (5%)
Very Low ($x \leq 52.5$)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
Total	25 (100%)	25 (100%)	20 (100%)	20 (100%)

4.3. The effectiveness of erroneous examples toward students' mathematical disposition

The inferential analysis revealed that through paired samples t-test, can be obtained information that there was insufficient evidence to say that learning mathematics from the erroneous examples in the individual setting (*p-value=0.2059>0.05*) and collaborative setting (*p-value=0.6835>0.05*) affect students' mathematical disposition. Furthermore, through one sample t-test, it can be found that there was sufficient evidence to conclude that the mean score of students' mathematical disposition in the individual setting is greater than 82.5 (*p-value=0.0039<0.05*). On the contrary, in a collaborative setting, there was no adequate evidence to say that the mean score of students' mathematical disposition after the intervention is greater than 82.5 (*p-value=0.6799>0.05*). Moreover, one proportion z-test showed that not more than 70% of students in the individual (*p-value=0.5118>0.05*) and collaborative setting (*p-value=0.9987>0.05*) whose mathematical disposition score is greater than 82.5 at the after the intervention. Thus, learning mathematics from the erroneous example in both the individual and collaborative setting is not effective for facilitating students' mathematical disposition. Some possible reasons behind these finding will be elaborated as follow.

There was a slight decreasing on the mean score of students' mathematical disposition (82.20 to 81.50) in a collaborative setting (Table 2). This might be caused by unequal distribution among groups as the students' prior knowledge and capability were not considered when assigning students to the group. In fact, the collaborative setting will be beneficial for students depends upon how well a collaborative setting is facilitated and students' conditions [36].

Learning material might also affect students' disposition. The learning material used in this study is the application of the first derivative; it is quite complex material to be learned. Moreover, finding and correcting error contained in the erroneous example is considered as a challenging task for students [38] since such task requires students to think logically, self-explain, and observe every single step in the example thoroughly. This may decrease students' willingness to put an effort for understanding a problem, procedures, concepts, or other aspects related to mathematics as indicated by the data from the questionnaire in the individual (70.50% to 69.00%) and collaborative setting (72.50% to 65.63%).

This study also found that students' average score in individual setting on the appreciation towards advantages and applications of mathematics aspect was increased (76.75% to 78.00%). On the other hand, it was decreased in the collaborative setting (73.75% to 71.25%). Nevertheless, actually, the change was not substantial. This may be caused there was very limited erroneous example related to the application of the first derivative in daily life that presented to the students. Although, Prabawanto [35] argues that presenting many problems related to daily life can lead students to be more appreciate to the advantages and applications of mathematics. Jensen [35] also states that one of the characteristics of mathematics learning that can support students' mathematical disposition improvement is that confronts students with the advantages of mathematics in daily life.

To sum up, even though some literature indicate that confronting erroneous examples to students may have a potential to facilitate students' mathematical disposition in both of individual and collaborative setting, this study found the different result. The other possible reason behind this result—besides the design of erroneous example, the topic of learning, the implementation of learning setting—is that mathematical disposition is a part of the attitude. Regarding the attitude, it cannot be changed in a short time [36]. It means that presented erroneous example to students just in four meeting was not enough to facilitate students' mathematical disposition.

5. Conclusion

This study found that design of erroneous example, the topic of learning, how the setting of learning is implemented, students' conditions, and time allocation may lead to the result that erroneous example in both individual and collaborative setting could not statistically significant in facilitating students' mathematical disposition. This study, however, notices that learning mathematics from the erroneous example in an individual setting increased the score of all aspects of mathematical disposition even though it is not significant. Furthermore, the erroneous example in individual and collaborative setting also improve their self-regulatory skills even though it is not significant. The implication for teaching and learning mathematics with the erroneous example is that teacher should aware towards the design of erroneous examples, topic of learning, the way to implement setting of learning optimally, and time allocation such that presenting erroneous example would be more beneficial for students. Nevertheless, this study is also limited to the participants that may make the result is not representative enough.

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